

Name: \_\_\_\_\_ Date: \_\_\_\_\_

**Master 3.3a Completing the Square**

Expand each binomial square.

$(x+1)^2 = x^2 + 2x + 1$	$(x-1)^2 = x^2 - 2x + 1$
$(x+2)^2 = x^2 + 4x + 4$	$(x-2)^2 = x^2 - 4x + 4$
$(x+3)^2 = x^2 + 6x + 9$	$(x-3)^2 = x^2 - 6x + 9$
$(x+4)^2 = x^2 + 8x + 16$	$(x-4)^2 = x^2 - 8x + 16$
$(x+5)^2 = x^2 + 10x + 25$	$(x-5)^2 = x^2 - 10x + 25$

What do you notice about the first term in each trinomial?

How is the constant in the binomial related to the middle term in the trinomial?

$x^2$   
= Doubling itself

How is the constant in the binomial related to the last term in the trinomial?

= Squared

Determine the number that must be added to each trinomial to form a perfect square trinomial, then factor the trinomial.

$x^2 + 14x + \boxed{49} = (x + \boxed{7})^2$

$x^2 + 20x + \boxed{100} = (x + \boxed{10})^2$

$x^2 - 16x + \boxed{64} = (x - \boxed{8})^2$

$x^2 - 24x + \boxed{144} = (x - \boxed{12})^2$

$x^2 + 22x + \boxed{121} = (x + \boxed{11})^2$

$x^2 - 5x - 5x + 25$   
 $x^2 - 10x + 25$

How did you determine the number to be added to complete the square?

$ax^2 + bx + c$   $(\frac{b}{a})^2$

How did you determine the value of the constant in the binomial square?

Determine the decimal that must be added to each trinomial to form a perfect square trinomial, then factor the trinomial

$x^2 + 3x + \underline{\hspace{1cm}} =$   $(\frac{3}{2})^2 = \frac{9}{4}$   $x^2 + 6x + \boxed{9} = (x+3)^2$

$x^2 - 7x + \underline{\hspace{1cm}} =$   $(-\frac{7}{2})^2 = \frac{49}{4}$   $x^2 - 14x + \boxed{49} = (x-7)^2$

$x^2 + x + \underline{\hspace{1cm}} =$   $(\frac{1}{2})^2 = \frac{1}{4}$   $x^2 + 2x + \boxed{1} = (x+1)^2$

$x^2 - 1.5x + \underline{\hspace{1cm}} =$   $(-\frac{3}{4})^2 = \frac{9}{16}$   $x^2 - 3x + \boxed{\frac{9}{4}} = (x-\frac{3}{2})^2$

**Master 3.3a**  
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3.3 Solving By Completing the Square

S.T.

Given  $x^2 - 12x - 7 = 0$

How many answers could the equation have? *two should*

*sum -12 ? = -7 Does Not Factor!*

If your life depended on finding those answers what is your problem? *Factoring does not work! I must use  $\sqrt{\quad}$  property*

Procedure for Completing the square:  $ax^2 + bx + c = 0$

1. If  $a \neq 1$  ~~make~~ make it = 1 (divide or multiply) *set a out (a)*
2. Move the constant to the other side of the equation
3. Determine the value of the number to complete the square.
4. Add that number to both sides of the equation.
5. Factor
6. Solve for x *&  $\sqrt{\quad}$  property)*

$x^2 - 12x - 7 = 0$

$x^2 - 4x - 1 = 0$

$x^2 - 12x + 36 = 7 + 36$   
 $\sqrt{(x-6)^2} = \sqrt{43}$

$x^2 - 4x + 4 = 1 + 4$   
 $\sqrt{(x-2)^2} = \sqrt{5}$

$x - 6 = \pm\sqrt{43}$   
 $x = 6 \pm \sqrt{43} \quad \checkmark$

$x - 2 = \pm\sqrt{5}$

$x = 6 \pm \sqrt{43}$

$x = 2 \pm \sqrt{5}$

Verify Calc.  $x = 6 + \sqrt{43}$  or  $6 - \sqrt{43}$   
 $= 12.56$        $- .56$

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Solving where  $a \neq 1$   $\left. \begin{matrix} S = -16 \\ P = 23 \end{matrix} \right\}$

$$4x^2 - 16x - 7 \neq 0$$

$$x^2 - 4x - \frac{7}{4} = 0$$

$$x^2 - 4x + 4 = \frac{7}{4} + 4 \frac{16}{4}$$

$$\sqrt{(x-2)^2} = \sqrt{\frac{23}{4}}$$

$$x-2 = \pm \sqrt{\frac{23}{4}}$$

$$x = 2 \pm \frac{\sqrt{23}}{2} = \frac{4 \pm \sqrt{23}}{2}$$

$\therefore -\frac{1}{3}x^2 - x + 4 = 0$

$$x^2 + 3x - 12 = 0$$

$$x^2 + 3x + \frac{9}{4} = 12 + \frac{9}{4}$$

$$\left(\frac{3}{2}\right)^2$$

$$\left(x + \frac{3}{2}\right)^2 = 14.25$$

$$x + \frac{3}{2} = \pm \sqrt{14.25}$$

$$x = -\frac{3}{2} \pm \sqrt{14.25}$$

$$x = \frac{-3 \pm \sqrt{57}}{2}$$

$$-3x^2 - 12x + 17 = 0$$

$$\frac{-3}{-3} \quad \frac{-12}{-3}$$

$$x^2 + 4x + \frac{17}{3} = 0$$

$$x^2 + 4x + 4 = \frac{17}{3} + 4$$

$$(x+2)^2 = \frac{29}{3}$$

$$x+2 = \pm \sqrt{\frac{29}{3}}$$

$$x = -2 \pm \sqrt{\frac{29}{3}} \quad \left\{ \begin{matrix} \sqrt{3} \text{ R+Den} \\ \sqrt{3} \end{matrix} \right.$$

$$x = -2 \pm \frac{\sqrt{87}}{3} = \frac{-6 \pm \sqrt{87}}{3}$$

Master 3.3b

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④  $3x^2 - 12x - 7 = 0$   $x - 2 \begin{matrix} \text{factor} \\ + - 2 \end{matrix} \text{ D.N.F.}$

$$3[x^2 - 4x + 4 - 4] - 7 = 0$$

$$3[x^2 - 4x + 4] - 12 - 7 = 0$$

$$3[x - 2]^2 - 19 = 0$$

$$\sqrt{3[x - 2]^2} = \sqrt{\frac{19}{3}}$$

$$x - 2 = \pm \sqrt{\frac{19}{3}}$$

$$x = 2 \pm \sqrt{\frac{19}{3}}$$

⑤  $4x^2 - 24x - 13 = 0$

$$4[x^2 - 6x + 9 - 9] - 13 = 0$$

$$4(x - 3)^2 - 36 - 13 = 0$$

$$4(x - 3)^2 - 49 = 0$$

$$4(x - 3)^2 = 49$$

$$\sqrt{4(x - 3)^2} = \sqrt{\frac{49}{4}}$$

$$x - 3 = \pm \sqrt{\frac{49}{4}}$$

$$x = 3 \pm \sqrt{\frac{49}{4}}$$

$$x = 3 \pm \frac{7}{2}$$

$$x = \frac{13}{2} \text{ or } \frac{-1}{2}$$

∴ could have been done with factoring

**Example 4****Solving a Problem Using a Quadratic Equation**

A football is kicked vertically. The approximate height of the football,  $h$  metres, after  $t$  seconds is modelled by this formula:  $h = 1 + 20t - 5t^2$ . When will the football hit the ground? Give the answer to the nearest tenth of a second.

$$h = 0$$

$$\infty 0 = 1 + 20t - 5t^2$$

