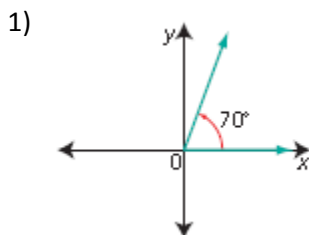


# Math 20-1 Year End Review Package Key

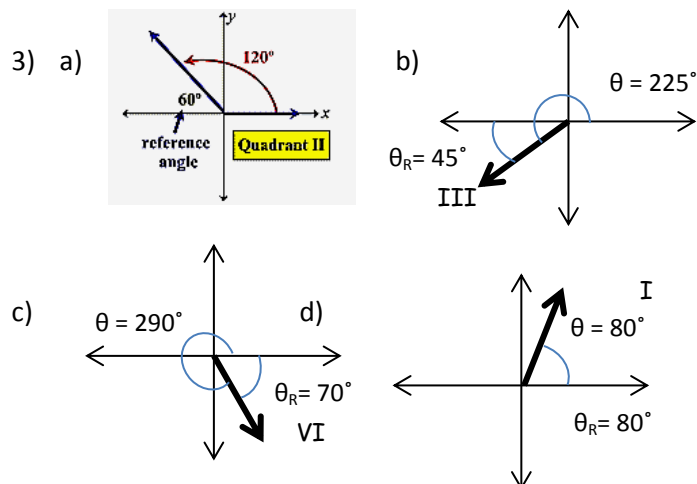
## Chapter 1 Sequences & Series

- 1) The value of  $n$  will be a natural number
- 2) Discrete. Values between the terms of the sequence are not included in the sequence.
- 3) A
- 4) 5, 7, 9
- 5)  $d = 4, t_n = 4n + 3$
- 6)  $d = 7, t_1 = -19$
- 7)  $n = 32$
- 8)  $S_{20} = 800$
- 9)  $S_{19} = 893$
- 10)  $n = 21$
- 11) a) common difference, b) y-intercept + slope value equals  $t_1$ .  
c) continuous, domain is all real numbers discrete, domain is natural numbers  
d) Continuous means that all values between known points are included. The points on the graph would be connected. Sequences are not continuous.
- 12)  $r = 3, t_n = 8(3)^{n-1}$
- 13) a) 9, 12, 15 b) 12, 24, 48
- 14)  $n = 10$
- 15)  $t_7 = \frac{25}{8}$
- 16)  $r = 3$ , first 5 terms: 2, 6, 18, 54, 162
- 17) \$38 906.14
- 18)  $S_{12} = \frac{102375}{512} \approx 199.951$
- 19)  $S_n = 2\,516\,583$
- 20)  $-1 < r < 1$
- 21) a) a convergent infinite series has a sum
- 22) a)  $r = \frac{1}{2}$ , convergent b)  $r = 2$ , divergent, no sum
- 23) 168,  $-\frac{336}{5}, \frac{672}{25}$
- 24) A  $\rightarrow$  graph on left, B  $\rightarrow$  graph on right
- 25) a) Height after 4<sup>th</sup> bounce is  $t_5 = 0.512$  m  
b) 46.4 m

## Chapter 2: Trigonometry



$$2) \sin \theta = \frac{y}{r} \quad \cos \theta = \frac{x}{r} \quad \tan \theta = \frac{y}{x}$$



- 4) a) Angle in standard position:  
 $30^\circ \rightarrow$  quadrant II  $150^\circ$ ,  
 $45^\circ \rightarrow$  quadrant III  $225^\circ$ ,  
 $60^\circ \rightarrow$  quadrant IV  $300^\circ$   
 b)  $\theta_R = 54^\circ, 126^\circ, 234^\circ, 306^\circ$
- 5) a) No. Reference angles in standard position are measured between the terminal arm and the nearest x-axis.  
 b)  $250^\circ, 290^\circ$   
 c)  $\theta_R = 70^\circ$
- 6) a) x-axis  
 b) The reference angles are the same for the related angles in standard position.  
 c) (3, -5) and (-3, -5)

7)

$\sin \theta$	$\cos \theta$	$\tan \theta$
$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{3}}$ or $\frac{\sqrt{3}}{3}$
$\frac{1}{\sqrt{2}}$ or $\frac{\sqrt{2}}{2}$	$\frac{1}{\sqrt{2}}$ or $\frac{\sqrt{2}}{2}$	1
$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$
1	0	Undefined
0	-1	0
-1	0	Undefined

$$8) \sin 150^\circ = \frac{1}{2} \quad \cos 120^\circ = -\frac{1}{2}$$

$$\tan 300^\circ = -\sqrt{3} \quad \sin 310^\circ = -0.76604$$

$$\cos 210^\circ = -\frac{\sqrt{3}}{2} \quad \tan 200^\circ = 0.36397$$

9) a) 41

b)  $\sin \theta = \frac{-9}{41}$ ,  $\cos \theta = \frac{40}{41}$ ,  $\tan \theta = \frac{-9}{40}$   $\theta = 347.3^\circ$

10)  $\sin \theta = \frac{5}{\sqrt{61}}$ ,  $\cos \theta = \frac{-6}{\sqrt{61}}$ ,  $\tan \theta = \frac{-5}{6}$   $\theta = 140.2^\circ$

11)  $\sin \theta = 0.615$ ,  $\theta = 37.95^\circ$  or  $142.05^\circ$ ,  
 $\tan \theta = -2.43$ ,  $\theta = 112.4^\circ$  or  $292.4^\circ$

$\tan \frac{-\sqrt{3}}{3} \theta = 150^\circ$  or  $330^\circ$

$\cos \theta = \frac{1}{\sqrt{2}}$   $\theta = 45^\circ$  or  $315^\circ$

$\sin \theta = \frac{-1}{2}$   $\theta = 210^\circ$  or  $330^\circ$

$\cos \theta = \frac{\sqrt{3}}{2}$   $\theta = 30^\circ$  or  $330^\circ$

12)  $45\sqrt{2}$

13) a)  $a = 7.36$

b) ambiguous case  $B = 72.9^\circ$  or  $B = 107.1^\circ$

c)  $c = 3.22$

d)  $B = 73.4^\circ$

14)  $A = 435 \text{ cm}^2$

15) 9.5 yards

16) angle =  $91.3^\circ$

17)  $BC = 16.03 \text{ km}$

**Chapter 3: Quadratic Functions**

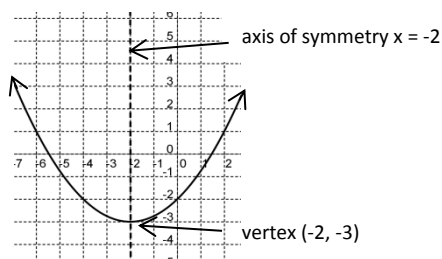
1)  $V(3, 1)$  Axis of Symmetry  $x = 3$   
 Horizontally translated by 3 units right and vertically by 1 unit up. Domain  $x \in \mathbb{R}$  Range  $y \geq 1$ .

$y = \frac{5}{9}(x-3)^2 + 1$

2) B

3) C

4) a) see diagram



b) x-intercepts  $-2 \pm \sqrt{3}$ , y-intercept -2

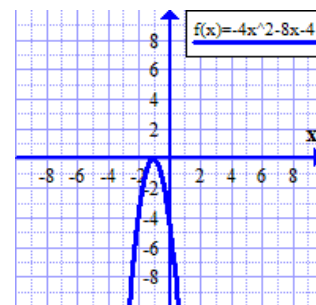
c)  $a = \frac{1}{4}$  → shape of parabola (wide than  $y = x^2$ )

a is positive → orientation (parabola opens up)

p is -2 → parabola shifts horizontally, 2 units left

q is -3 → parabola shifts vertically, 3 units down

5)  $V(-1, 0)$  A of S  $x = -1$   
 x-intercept -1  
 y-intercept -4

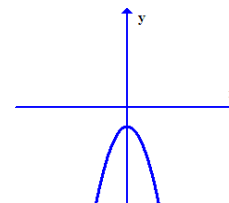


6) vertex  $(-4, -36)$ , axis of symmetry  $x = -4$ ,  
 opens up, minimum value  $y = -36$ ,  
 x-intercepts -10 and 2, y-intercept -20  
 Domain =  $\{x | x \in \mathbb{R}\}$  Range  $\{y | y \geq -36, y \in \mathbb{R}\}$

7)  $y = -\frac{3}{2}(x-2)^2 + 6$

8) C

9) Vertex at  $(0, -b)$   
 see diagram



10)  $y = 9(x+5)^2 + 6$

11)  $y = 2x^2 + 4x - 1$

12)  $y = 2(x^2 - 6x + 9) - 11$

$y = 2(x^2 - 6x + 9) - 18 - 11$

$y = 2(x-3)^2 - 29$   $V(3, -29)$

13) a)  $V(0, 4)$  b)  $V(5, 0)$

14)  $y = -2x^2 + 2x + 4$

15)  $x = 1$

16) parabola shifts 6 units right and 3 units down

17) a)  $y = (x+3)^2 + 6$  vertex  $(-3, 6)$

range  $\{y | y \geq 6, y \in \mathbb{R}\}$ , no zeros

b)  $y = -3(x+6)^2 + 8$  vertex  $(-6, 8)$

range  $\{y | y \leq 8, y \in \mathbb{R}\}$ , 2 zeros

18) y coordinate of vertex is -8

19) A

20) C

21) B

22) B

23) C

24) a)  $y = -5x^2 + 70x + 1200$  or  $y = (120 - 5x)(10 + 1x)$

b) best price \$17, maximum revenue is \$1 445

25) maximum area is  $216 \text{ cm}^2$

26) maximum area is  $1800 \text{ m}^2$

27) product is  $(10)(-10) = -100$

28) 72%

29) 5m x 25m

## Chapter 4: Quadratic Equations

- a)  $(x-3)(x-9)$  b)  $(2x-1)(2x+5)$   
c)  $-(x+5)(x-3)$  d)  $(x-5y)(x+5y)$   
e)  $(2x-9y)(2x+9y)$  f)  $(2x+5)((x+2)$   
g)  $-4(2x+3)(x+3)$  h)  $(5x-11y-26)(5x+11y-4)$
- factors are  $(3y+2)(2y-3)$   $k = -5$
- $h = 2$
- $x = -3.33$  or  $x = -3$ , zeros
- $x = -2$  or  $x = -\frac{3}{4}$
- $x = 5$  or  $x = -5$
- $x = 9$  or  $x = -5$
- $x = -2$  or  $x = -16$
- $x = -3 + \sqrt{10}$  or  $x = -3 - \sqrt{10}$
- $x = \frac{12+\sqrt{7}}{4}$  and  $\frac{12-\sqrt{7}}{4}$
- Discriminant is  $-76$ . There are no real roots.
- a) no real roots  
b) 2 distinct real roots
- a) 2 distinct real roots  
b) 2 distinct real roots  
c) 2 equal real roots  
d) 2 distinct real roots  
e) no real roots
- $k < -7$
- $x = -\frac{1}{2}$
- $x^2 - 3x - 10 = 0$  or  $a(x^2 - 3x - 10) = 0$  where  $a$  is a number.
- 3.12 seconds
- The numbers are 3 and 12.
- 21 m x 12 m
- C
- length of the photo is 28.97 cm.

## Chapter 5: Radical Expressions and Equations

- 1, 4, 9, 25, 36, 49, 64, 81, 100, 121, 144, 169
- 1, 8, 27, 64, 125
- $-\sqrt{25}$ ,  $\sqrt[3]{32}$ ,  $\sqrt{7}$ ,  $\sqrt[3]{27}$ ,  $2\sqrt{3}$ ,
- True  $(-x)^2 = x^2$ ,  $\sqrt{x^2} = |x|$ ,  $\sqrt{9^2} = 3$
- C
- a)  $2\sqrt{2}$  b)  $3\sqrt[3]{2}$  c)  $2a^3b\sqrt{3b}$ ,  $a \geq 0, b \geq 0$   
d)  $2\sqrt{6}$  e)  $-2x^3\sqrt{6y^2}$

- a)  $-\sqrt{200}$  b)  $\sqrt{24}$  c)  $\sqrt[3]{54y^5}$   
d)  $\sqrt{18}$  e)  $\sqrt[3]{32}$  f)  $\sqrt[4]{a^4b^5c^4}$
- a)  $x \geq 0$  b)  $x \geq -4$  c)  $x \geq 3$  d)  $x \geq -\frac{5}{2}$
- a)  $10\sqrt{11}$  b)  $3\sqrt{5}$   
c)  $-9\sqrt{2} + 6\sqrt{2x} - 4x\sqrt{2x}$ ,  $x \geq 0$
- a)  $6\sqrt{5} + 8\sqrt{10}$  b)  $-8\sqrt[3]{54}$  c)  $12 + 24\sqrt{6}$   
d)  $-21 + 3\sqrt{3}$  e)  $63\sqrt{2} - 35\sqrt{21} - 36 + 10\sqrt{42}$
- a)  $\sqrt{2}$  b)  $\frac{2\sqrt{15}}{3}$  c)  $\frac{27+9\sqrt{7}}{2}$   
d)  $-\frac{\sqrt{6}}{24}$  e)  $\frac{12-3\sqrt{5}}{11}$
- a)  $1 \pm 2\sqrt{5}$  b)  $3 \pm 2\sqrt{6}$  c)  $2 \pm 3\sqrt{3}$
- conjugates, conjugate binomials, rational
- a)  $7\sqrt{2}$  b) 14
- a)  $x = 6$ ,  $\{x \geq -3\}$  b)  $x = 6$ ,  $\{x \geq -3\}$  c) no solution  
d)  $x = 1$ ,  $\left\{x \geq \frac{1}{4}\right\}$  e)  $x = 1$ ,  $\{x \geq 1\}$   
f)  $x = 23$ ,  $\{x \geq -2\}$
- a) 1.90 minutes b) 223.4 m

## Chapter 6: Rational Expressions and Equations

- $\{x \mid x \neq -3, x \in \mathbb{R}\}$
- a) 15 b)  $2(x-1)(x+3)$  or  $2x^2 + 4x - 6$
- a)  $\frac{4x}{x-3}$ ,  $x \neq 3$  b)  $\frac{x-2}{x+3}$ ,  $x \neq -3, -5$   
c)  $\frac{x^2 - x - 30}{(x+2)(x+1)}$ ,  $x \neq -5, -3, -2, -1$ ,  
d)  $\frac{x-9}{(x-8)(x-7)}$ ,  $x \neq 7, 8$  e)  $\frac{x-2}{x+5}$ ,  $x \neq \pm 5, -2$   
f)  $\frac{2x-1}{(x-3)(x+2)(x+4)}$ ,  $x \neq -4, -2, 3$   
g)  $\frac{4xn}{3}$ ,  $m, n, x, y \neq 0$  h)  $\frac{11}{4x+12}$ ,  $x \neq -3$
- a)  $x = 0$ ,  $x \neq 1$  b)  $x = \frac{3}{2}$  or  $2$ ,  $x \neq -2, -1$   
c)  $x = 10$ ,  $x \neq 0$  d)  $x = 3$  or  $4$ ,  $x \neq \pm 1$
- $\frac{100}{9}$  or 11.1 minutes
- 12 Km/h

7) Superman 116.9 Km/h Flash 136.9 Km/h

**Chapter 7: Absolute Value and Reciprocal Functions**

$-4\frac{1}{2}, -0.1, |-0.01|, |-6.75|, 7, \left|\frac{-22}{3}\right|, |-8.5|$

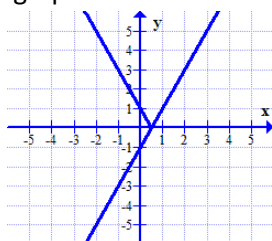
1)  
2) a) -9 b) -25

3) possible ages are  $15\frac{1}{4}$  or  $16\frac{3}{4}$

4) D  
5) A  
6)

x	f(x)
-3	10
0	5
3	0
6	5

7) graph



	f(x)	g(x)
x-int	$\frac{1}{2}$	$\frac{1}{2}$
y-int	-1	1
domain	$x \in \mathfrak{R}$	$x \in \mathfrak{R}$
range	$y \in \mathfrak{R}$	$\{y \geq 0\}$
set of invariant points	$\left\{x \geq \frac{1}{2}\right\}$	

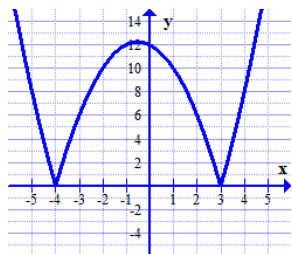
8) graph

x-int -4 or 3  
y-int 12

domain  $x \in \mathfrak{R}$

range  $y \geq 0$

set of invariant points  $\{-4 \geq x \geq 3\}$



9) a)  $|x+5| = \begin{cases} x+5, & x \geq -5 \\ -(x+5), & x < -5 \end{cases}$

b)  $|x-3| = \begin{cases} -x-3, & x \leq -3 \\ -(-x-3), & x > -3 \end{cases}$

9c)  $|x^2+9| = \begin{cases} -x^2+9, & -3 \leq x \leq 3 \\ x^2-9, & -3 > x > 3 \end{cases}$

d)  $|x^2+5x+4| = \begin{cases} x^2+5x+4, & -4 \geq x \geq -1 \\ -(x^2+5x+4), & -4 < x < -1 \end{cases}$

10) a)  $x = -12$  or  $2$  b)  $x = \frac{1}{2}$  or  $5$

c)  $x = 4 \pm 2\sqrt{6}$  d) no solution

11) a)  $x = \frac{2}{7}$  b)  $x = -4, x = 3$

12) The x-intercept of the linear graph becomes the position of a vertical asymptote on the graph of the reciprocal.

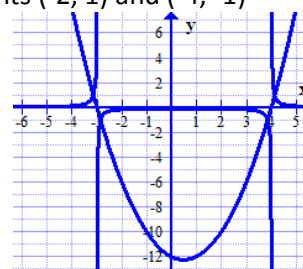
13) The equations for the vertical asymptotes are equal to the value; the equations for the non-permissible values are not equal to the value.

Asymptote Equations:  $x = 7$  and  $x = -4$

Non-permissible values  $x \neq 7$  and  $x \neq -4$

14) invariant points (-2, 1) and (-4, -1)

15) graph



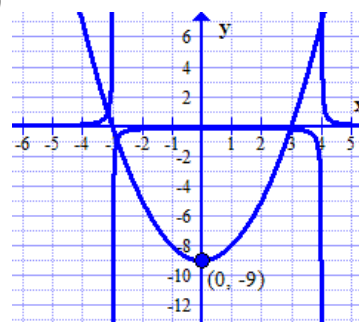
16) a) domain  $h(x): x \in \mathfrak{R}$

domain  $j(x): \{x | x \neq 2, x \in \mathfrak{R}\}$

b) range  $h(x): y \in \mathfrak{R}$

range  $j(x): \{y | y \neq 0, y \in \mathfrak{R}\}$

17)



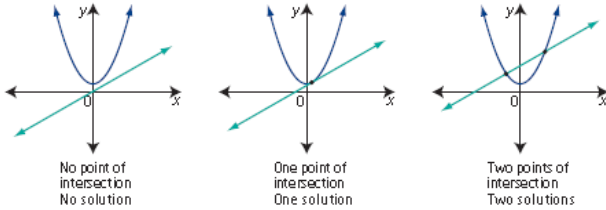
## Chapter 8: Systems of Equations

1) (1, 3) is not a solution, (4, 0) is a solution

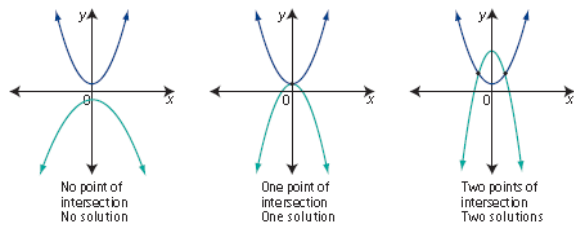
2) a)  $x = -1$  and  $y = 8$  or  $x = 0$  and  $y = 1$

b) no solution

3)



4)



5) a)  $x = -2$  and  $y = 7$

b)  $x = -4$  and  $y = 9$  or  $x = 4$  and  $y = -7$

6) a) perimeter  $y = 8x + 10$ , area  $3y = 3x^2 + 15x$

b)  $x = -2$  or  $x = 5$

c)  $x$  must be a positive value,  $x \neq -2$

d)  $x = 5$  dimensions are 15 x 10, perimeter is 50, area is 150

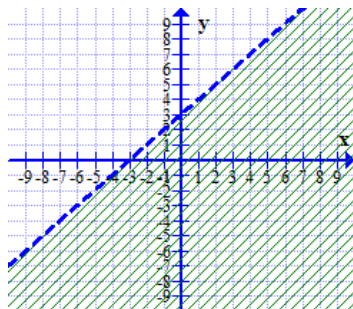
7) a)  $x^2 + y^2 = 74$

b)  $x = \pm 7$  and  $y = \pm 5$

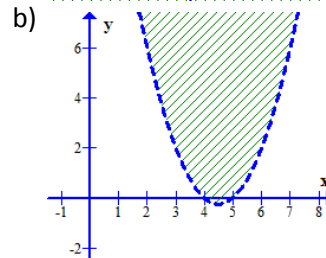
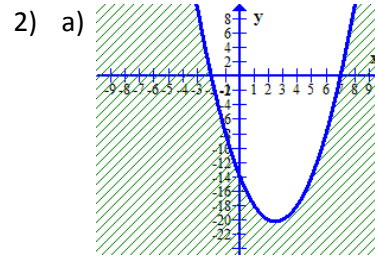
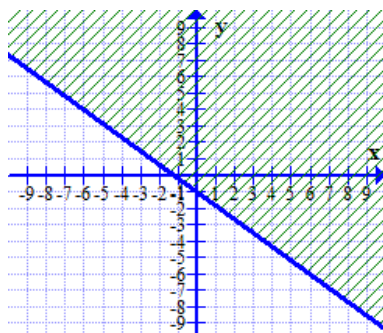
$x^2 - y^2 = 24$

## Chapter 9: Linear and Quadratic Inequalities

1) a)



b)



3) a) set notation  $\left\{x > \frac{5}{2}\right\}$  or

interval notation  $\left(\frac{5}{2}, \infty\right)$

b) set notation  $\left\{2 - \sqrt{2} < x < 2 + \sqrt{2}\right\}$  or

interval notation  $\left(2 - \sqrt{2}, 2 + \sqrt{2}\right)$

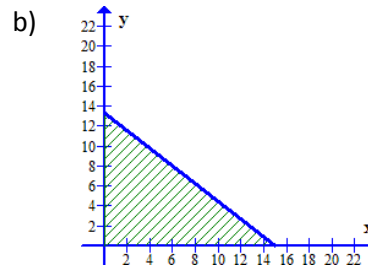
4) a) set notation  $\{x \geq -5\}$  or

interval notation  $[-5, \infty)$

b) set notation  $\left\{-\frac{5}{3} \geq x \geq -1\right\}$  or

interval notation  $\left[-\frac{5}{3}, -1\right]$

5) a)  $x$  is number of flats of marigolds,  $y$  is number of flats of petunias  $8x + 9y \leq 120$



c)  $x$  and  $y$  must be whole numbers

$\{0 \leq x \leq 15\}$  and  $\{0 \leq y \leq 13\}$

d) any whole number ordered pair in shaded solution region.

6) a)  $11x + 6y \leq 600$

b)  $x$  and  $y$  must be whole numbers

$\{0 \leq x \leq 54\}$  and  $\{0 \leq y \leq 100\}$

7) a) Price \$7.50 b) between \$6 and \$9

